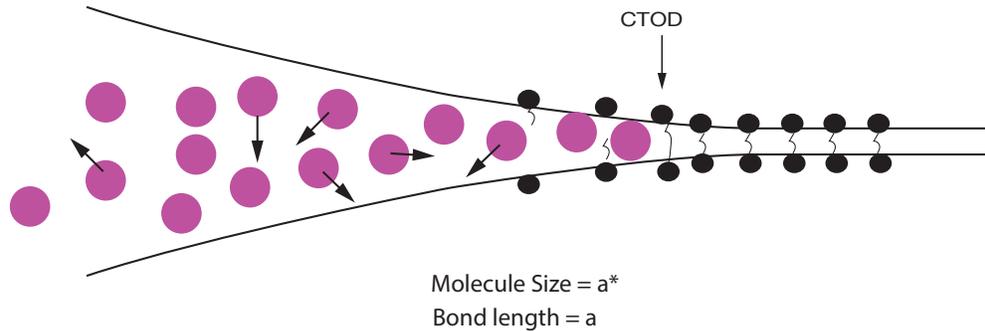
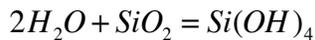


3F: Subcritical Crack Growth



Definition:

- The crack propagates at a $K_I < K_{IC}$ (it is subcritical!)
- What is K_{IC} is the stress intensity factor at which the crack propagates catastrophically because the critical condition for fracture in terms the work of fracture or the crack-tip criterion for fracture is met.
- How can the crack grow below the critical condition? Consider crack propagating in glass in the presence of water which is a corroding species. Now, the bond can be broken by a chemical reaction between the environment and the bonds at the crack tip. For example,



What are the expectations about the fracture process?

- i. The crack propagation will become time dependent.
- ii. The corroding molecule must physical access to the crack tip. For example, if the CTOD is less than diameter of this molecule then the crack will not grow. As a result a lower value of K_I can be specified below which subcritical crack growth will not occur.
- iii. This lower value for stress-corrosion threshold is called K_{th} , which of course less than K_{IC} , but which depends on the size of the corrosive molecule.
- iv. Question then is how what is the velocity of the crack when $K_{th} < K_I < K_{IC}$.

Another name for this phenomenon is stress-corrosion. Other examples of stress-corrosion cracking are:

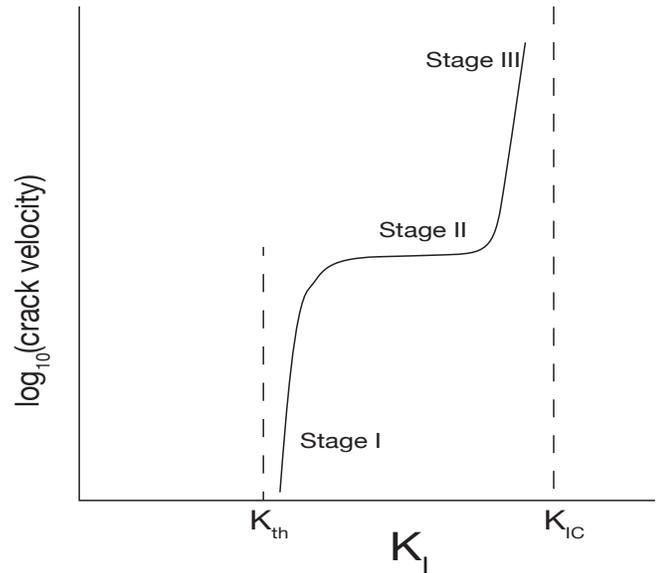
- Hydrogen embrittlement of steel. Hydrogen attacks the crack tip in iron causing the crack to grow subcritically.
- Corrosion of aluminum with acids.

*Environmental cracking

Typical sub-critical crack growth behavior

Laboratory data are plotted in the manner shown in the figure on the right. The crack velocity is shown in logarithmic scale since it can vary over several orders of magnitude.

We note that the crack has finite velocity when $K_{th} < K_I < K_{IC}$



Crack growth velocity consists of three stages (taking water corrosion of glass as the example)

Stage I

Crack velocity approaches zero (asymptotically) as $K_I \rightarrow K_{th}$

The design criterion in situations where stress-corrosion cracking is possible is the $K_I < K_{th}$ (as in hydrogen embrittlement of storage tanks made from steel)

Stage II

During this range ($K_{th} < K_I < K_{IC}$) the crack velocity is nearly independent of the stress intensity factor.

The molecule must travel from the environment to the crack tip in order to cause stress corrosion.

The transport of the molecule along the surface of the crack depends on chemical interactions between the surface and the molecule.

Therefore, the rate at which the molecules reach the crack-tip is controlled by surface chemistry rather than by the physical access to the bond at tip of the crack, which depends on the size of the molecule and the CTOD.

Stage III

The final stage in this scenario is where K_I begins to approach K_{IC} , then the crack is near the critical point and will travel quickly like in brittle fracture when $K_I = K_{IC}$

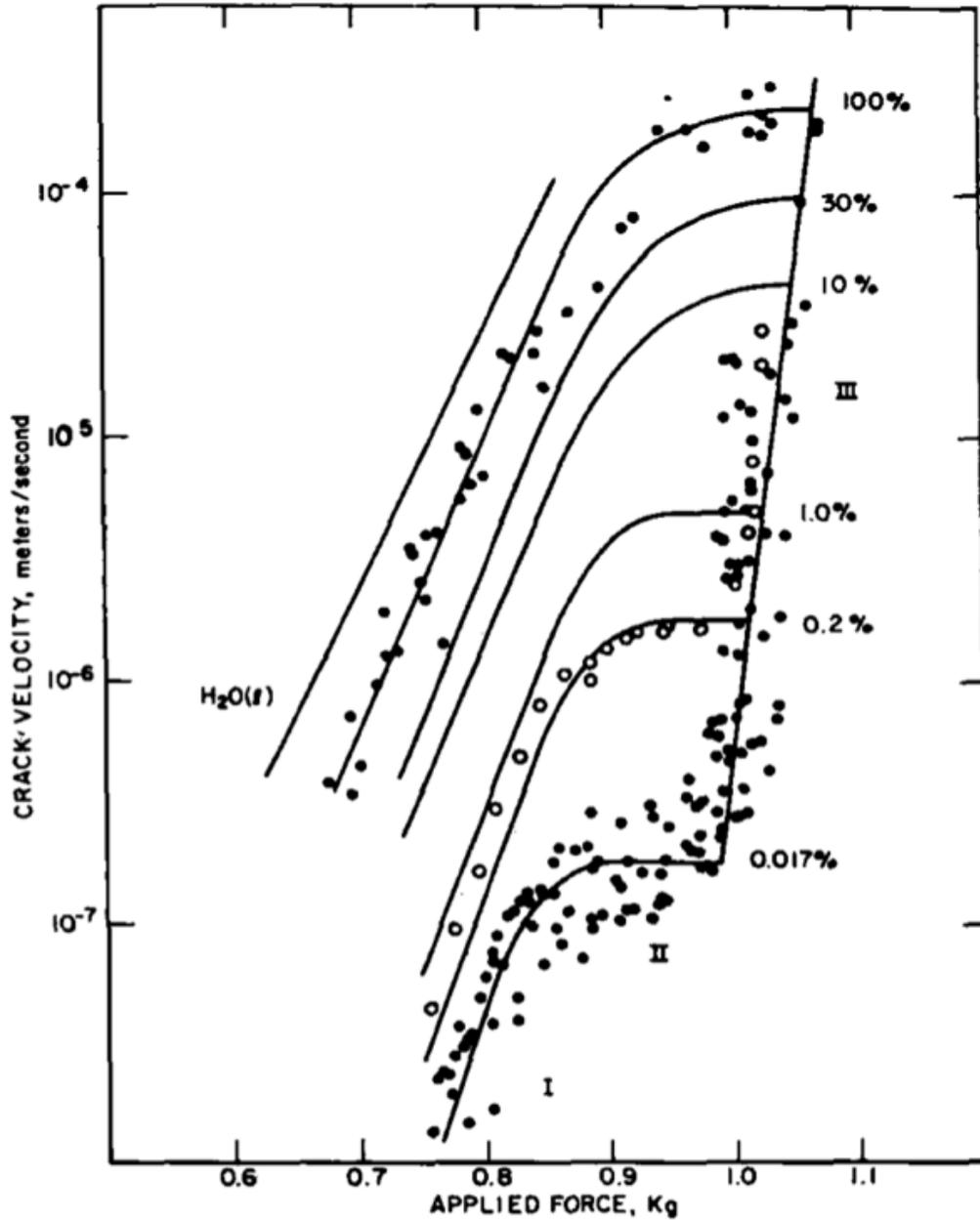


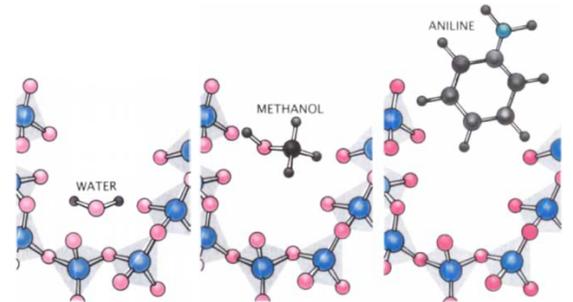
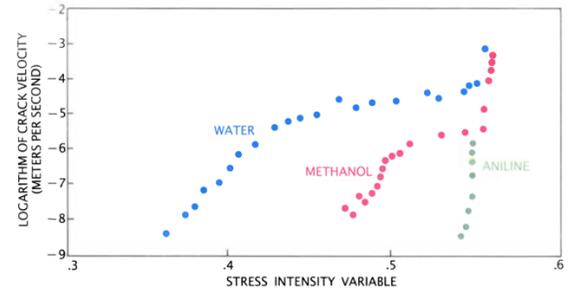
Fig. 3. Dependence of crack velocity on applied force. The percent relative humidity for each set of runs is given on the right-hand side of the diagram. Roman numerals identify the different regions of crack propagation. Region III shows only about half the points used to determine the line. The line, therefore, does not seem to fit the data as well as it would if all data in region III were plotted.

The data sets show the behavior show the crack velocity in silica glass with increasing humidity.

The trends for Stage I, II and III are to be noted.

Table II. Molecular Diameters of Test Chemicals

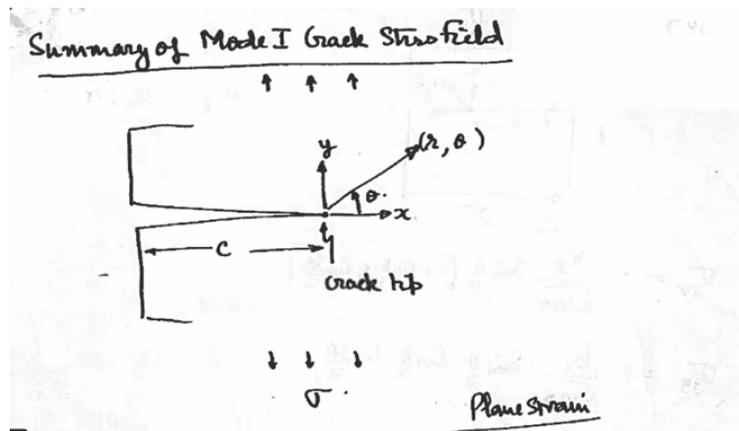
Chemical	σ (nm)
Water	0.265
Methanol	0.359
Ammonia	0.260
Hydrazine	0.36
Formamide	0.40
N-Methylformamide	0.45
n-Propylamine	0.46
tert-Butylamine	0.52
Aniline	0.55



MOLECULAR SIZE of a chemical substance affects its ability to speed the growth of cracks in glass. Water, which has a molecular size of only .26 nanometer (billionth of a meter), causes cracks to grow much faster than methanol (.36 nanometer) does, and aniline (.42 nanometer) has hardly any effect (top). In essence, water can readily enter a crack opening (which has a diameter of .4 to .5 nanometer), methanol has difficulty getting in and aniline is so big that it never arrives at the sites of bond rupture (bottom).

What is of note is that as the chemical species becomes larger the (apparent) threshold stress intensity factor becomes larger as well.

Therefore, it should be possible to predict the threshold value of the stress intensity factor for a given size of the chemical species.



$$K_I = \sigma \sqrt{\pi c}$$

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} (1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2})$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} (1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2})$$

$$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}$$

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) \text{ for Plane strain}$$

$$u_x = \frac{K_I}{G} \sqrt{\frac{2}{\pi r}} \sqrt{\frac{2}{2\pi}} \cos \frac{\theta}{2} (1 - 2\nu + \sin^2 \frac{\theta}{2})$$

$$u_y = \frac{K_I}{G} \sqrt{\frac{2}{\pi r}} \sin \frac{\theta}{2} (2 - 2\nu - \cos^2 \frac{\theta}{2})$$

$$u_z = 0$$

The equation that relates K_{th} to the size of the corrosive molecule is obtained from the equation for the displacement in the y direction in front of the crack tip which is given by

$$u_y = \frac{K_I}{G} \sqrt{\frac{\Omega^{1/3}}{2\pi}} \sin \frac{\theta}{2} (2 - 2\nu - \cos^2 \frac{\theta}{2}) \quad (1)$$

where r has been replaced by interatomic distance ($\Omega^{1/3}$). The relationship between the size of the corroding molecule and the value of the threshold stress intensity factor is given by

$K_I = K_{th}$, when $2u_y = a$ where a is the molecule size. Note the factor of 2 in the displacement because Eq. (1) give the displacement for one of the two faces of the crack tip.

Note that $\theta = \frac{\pi}{2}$ since we are interested in the displacement perpendicular to the plane of the crack.

Recall also that $G = \frac{E}{2(1+\nu)}$. Inserting into Eq. (1) we obtain

$$a = K_{th} \frac{4(1+\nu)}{E} \sqrt{\frac{\Omega^{1/3}}{2\pi}} \sin \frac{\pi}{2} (2 - 2\nu - \cos^2 \frac{\pi}{2}) \quad (2)$$

Substituting (from the previous lecture)

$$E = 73 \text{ GPa}$$

$$\nu = 0.15$$

$$\Omega^{1/3} = 3.35\text{E-}10 \text{ m}$$

$$a \text{ (water)} = 0.265 \text{ nm}$$

$$\sin \frac{\pi}{2} = \cos \frac{\pi}{2} = \frac{1}{\sqrt{2}}$$

E	7.300E+01GPa
	7.300E+10Pa
a	2.650E-01nm
nu	1.500E-01
sin/cos	7.072E-01
Omega^1/3	3.350E-10m

1st term	2nd term	3rd	result
6.301E-11	7.303E-06	1.061E+00	4.881E-16

K_th	5.429E+05Pa sqrt(m)
	5.429E-01MPa sqrt(m)

The threshold value of 0.54 MPa $\text{m}^{1/2}$ lies within a factor of two of the data given above.

It is also possible to compare the relative values of (approximate) K_{th} given in Eq. (2) by noting the prediction that the threshold stress intensity is proportional to the size of the corrosive molecule

	Mol size	K_{th}	Ratio relative to water	
			size	threshold
water	0.265	0.350	1.000	1.000
methanol	0.359	0.475	1.355	1.357
aniline	0.550	0.545	2.075	1.557